Exam 21 Name: ______________________________ RHS

Linear Algebra

Part I – 95 pts

1. Find the complete solution for
   $x - y + z = 1$
   $3x + 2y + z = 6$
   $4x + y + 2z = 7$

2. Find a 2 by 3 system $Ax = b$ whose complete solution is
   $$x = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} + a \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}$$

3. State the number of independent column vectors, using column operations.
   $$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 2 & 2 & 6 \\ 3 & 4 & 5 & 12 \end{bmatrix}$$

Part II – 10 pts

4. Find a 2 by 3 system $Ax = b$ whose complete solutions are sets of three consecutive integers, spanning set of integers.

Exam 22 Name: ________________________________ RHS
Linear Algebra

Part I – (95 pts)

1) State the matrix for the given transformation and its result.
   a. (1, 3) after rotation by 60°.
   b. (1, 3) after reflection over $y = x/\sqrt{3}$.

2) What 3 by 3 matrix represents the transformation that reflects every vector through $x - z$ plane?

3) What 2 by 2 matrix transform (1, 2) to (3, -2) and (3, 1) to (2, 5)?

4) Find the lengths and inner product of $v = (1, 2, 0, -1)$ and $w = (2, -1, 4, 0)$. Are the vectors orthogonal? Explain.

Part II – 10 pts

5) What 3 by 3 matrix represents the transformation that reflects every vector through a plane, $y = x$, followed by another plane, $z = x$?

Exam 23 Name: ____________________________________ RHS
Linear Algebra

Show your work for full credits.

Part I – 95 pts

1. For the given matrix A, find the orthogonal complement of
a) column space
b) row space

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & -2 & 0 \\ 4 & 4 & 4 \end{bmatrix}$$

2. If A and B are orthogonal subspaces, show that the only vector they have in common is the zero vector.

3. Find the projection vector of a on b, where

$$a = \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \ b = \begin{bmatrix} 6 \\ -1 \end{bmatrix}$$

4. What is the measure of angle between the given vectors?

$$a = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \ b = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

Part II – 10 pts

5. Are the planes 3x – y + z = 0 and x + 4y + z = 0 orthogonal subspaces? Explain.